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APPLICATION OF OPERATIONS RESEARCH TO INTERFERENCE

TECHNICAL NOTE NO. 1

ARMOUR RESEARCH FOUNDATION
of Illinois Institute of Technology
Technology Center
Chicago 16, Illinois

Report No. ARF 5177-TN-1

Contract No. AF 30(602)-2667

Prepared for
ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

Griffiss Air Force Base, New York

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UNCLASSIFIED REPORT

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ABSTRACT

A summary of work performed during the first six months of the program is presented, encompassing a review of several interference control methods and optimization techniques; selection of a control method for further investigation; development of applicable optimization techniques; and the design of a field test to check the assumptions and input data accuracies required for the control method. Controlling the operational parameters of frequency, transmitter power, geographical location and antenna orientation has been considered, with frequency assignment the interference control method selected for further study. The mathematical tools investigated in relation to their applicability to the control methods considered are linear programming, dynamic programming, and network and combinatorial search techniques. Emphasis is placed on radar interference problems and a field test using three L-band radars is described. Sample interference minimization problems with frequency as the controlled factor are presented, as well as the data available from the initial portions of the field test.

Respectfully submitted,

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PUBLICATION REVIEW

This report has been reviewed and is approved.

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I. OBJECTIVES

A. Objectives of the Project

This project is concerned with the application of techniques of operations research to problems of multiple interference encountered in congested electromagnetic environments. The specific purpose of the program is to investigate an interference control or reduction method (essentially an operational method), select appropriate mathematical techniques for optimizing with respect to the operational parameters under control, and then to test the assumptions and required accuracy of the input data for the control method by means of a field test. The operational parameters whose control has been considered include frequency, transmitter power, equipment location and antenna orientation. Among the mathematical tools of operations research under investigation in relation to their potential applicability to the control methods being considered are linear programming, dynamic programming and network methods. In pursuit of the objective emphasis is being placed on radar interference problems and field testing at the Verona Test Site.

B. Purpose of the Technical Note

This technical note is a summary of the work performed on the project during its first six months, from 9 April 1962 to 9 October 1962. It should be pointed out that because of a delay in transmitting the contract, effort on the program did not begin until 15 May 1962, so that work has been in progress somewhat less than five months.

II. ACCOMPLISHMENTS

Effort on the project has been devoted to the following tasks up to the present time:

1. Review of interference control methods, including an examination of various control methods available, optimization techniques applicable to these control methods, and possible field test situations which could be implemented to validate the various control methods.
2. Selection of a control method to be field tested, and the tailoring of an optimization technique to the control method selected.
3. Design and planning of a field test for the selected control method.
4. Initiation of the field test at the Verona Test Site.

III. REVIEW OF INTERFERENCE CONTROL METHODS, OPTIMIZATION TECHNIQUES AND VALIDATION TESTS

A. Review of Interference Control Methods

The concept of considering interference problems from the standpoint of operations research has implicit in it the key notion of optimization of a certain class of complex systems by exercising control over one or more operational variables of the equipments in the systems being optimized. These systems consist of populations of emitters and receptors of electromagnetic radiation whose simultaneous operation in a given environment may result in undesired interactions, that is, interference. It is significant that a system may in fact be formed because of unintended interactions even though there is no purposeful functional relationship among the component elements.

There are a great many possible means for preventing, reducing or eliminating interference among equipments in a given environment. If these means are restricted to operational methods applicable to large systems, they might be roughly grouped into three categories, according to the fundamental variable which the method is intended to modify. The three variables in question are time, frequency and space.

The first of these three categories consists of those methods for combating interference which affect the time distribution of electromagnetic energy. Included in this category are such well known techniques as time-sharing or time multiplexing of different messages in the same frequency channel, and regulation of time-on-air of different transmitters using the same or different frequency channels. Also in this category are those pulse modulation and demodulation techniques that exist primarily in the time domain. They incorporate such means for discriminating against undesired signals as for instance, prf discrimination, pulse position modulation used with an error correcting code, or demodulation circuits employing pulse width discrimination.

The second category encompasses those methods for reducing interference which affect the frequency distribution of electromagnetic energy. Prominent in this category are frequency assignment schemes, frequency-multiplexing and frequency diversity techniques.

The third category includes all those methods for combating interference which affect the spatial distribution of electromagnetic energy, in other words, the magnitude of the electromagnetic fields at any point in space. Methods such as the use of directional antennas or control over the gain pattern of an antenna, as well as every type of shielding, fall into this category. Included is also the method of minimizing transmitter power output.

Among the methods considered for testing on the current program are transmitter power assignment, geographical location, antenna pattern control and frequency assignment. These four methods were singled out for consideration for the following reasons: (1) the possibility of obtaining operational control over the pertinent variables involved, (2) the applicability of optimization techniques of operations research to these methods, and (3) the experience gained during the first year of the program in devising mathematical approached to the optimization problems posed by the use of these methods.

1. Transmitter Power Assignment

A great amount of experience was gained on the previous program, with the method of transmitter power assignment, both in fitting a mathematical optimization technique, namely, linear programming, to the interference control method, and in dealing with the problems encountered in putting the method into practice. Full details and a complete description may be found in the Final Report of the first year's effort on the program.[1] It will suffice here to describe the method very briefly.

Interference reduction is achieved by minimizing the power radiated by each transmitter in the system under control consistent with the constraint imposed by the requirement that the receiver signal-to-noise ratios are maintained above specified minimum values over the desired transmission channels. The effect of minimizing transmitter powers is to decrease the amount of electromagnetic energy delivered to the environment, and therefore to diminish the likelihood of interference, both to receivers in the system and to other receivers in the environment.

The criterion used in making an optimum power assignment is that the sum of the powers radiated by each of the transmitters in the system under control shall be a minimum. This implies a tradeoff between two quantities, one of which is transmitter power. The other quantity involved is the signal-to-noise plus interference ratio at each receiver in the system. It is the minimum acceptable value of this quantity for each receiver which determines the power required. Once this quantity has been chosen for each receiver, the smallest sum of transmitter powers sufficient to satisfy these requirements can be found.

In the simplest case, involving only one transmitter and one receiver, there is no interference to be taken into account, and hence the minimum power is determined solely by the propagation loss between the transmitter and receiver and the noise level at the receiver. However, as soon as other transmitters are added to the system there may also be interference at the receiver in question, and then the minimum required power of a transmitter desiring to transmit information to this receiver is a function of the strength of interfering signals as well as the receiver noise level and the propagation loss. Since the strength of interfering signals depends on the power radiated by each of the interfering transmitters and the propagation losses between them and the receiver in question, the minimum power which can be assigned the desired transmitter is a function of the power of each transmitter in the system whose signal affects the output of the specified receiver, the propagation losses between the receiver and each of these transmitters and the noise level at the receiver.

The same is true for every receiver in the system. Thus, by properly combining all the factors involved, the power assignment for the system which will minimize the total power radiated into the environment may be found. Clearly, the application of this method for controlling interference is not restricted to homogeneous systems, but may be applied to any complex of transmitting and receiving equipment for which the required information is available. Thus, not only complexes of different types of equipment in different frequency bands, but also mixtures of radar and communications equipment might be treated in this way, provided, of course, that the characteristics and requirements of each piece of equipment are known.

The technique developed for finding the optimum transmitter power assignment uses linear programming to arrive at the solution. The input parameters which must be known to utilize this technique are:

1. the system configuration; i. e., the geographical locations of the equipment, and the desired transmission paths,
2. the transmission coefficient between each transmitter and receiver in the system,
3. the noise level, ambient plus internal, at each receiver, and
4. the minimum acceptable signal-to-noise plus interference ratio for each receiver.

Included in the transmission coefficients are all the factors that determine the amount of signal accepted by a receiver from a given transmitter, such as antenna gain, propagation losses, transmission cable losses, and the transmitter and receiver spectrum signatures.

The solution is in the form of the power assigned to each transmitter in the system at its fundamental frequency. The sum of of the radiated powers is then the minimum possible, given the constraints imposed by the specified receiver signal-to-noise plus interference ratios.

The method of assigning transmitter power may be applied to both radar and communications equipment. Use of this method is certainly more obvious in communications systems than in systems involving radar equipment, however, there are situations involving radar in which this method appears useful. An example of one type of situation involving radar in which control over transmitter power may be applicable is one where the maximum detection range of interest to search radars and minimum target cross-section are known. Such a situation may occur in air traffic control, where the minimum target cross-section is known, and where the sector and maximum range to be covered by a particular radar at any given time may be well defined.

2. <u>Geographical Location</u>

An interference control method which depends on the geographical location of equipments to minimize interference implies the use of deployment as an independent system parameter in analysis. In this context, frequency assignments, transmitter powers, receiver sensitivities, antenna patterns and orientations, etc., must be known or be determinable from a system configuration. These latter parameters, however, provide other methods of exercising operational interference control, the implementation of each depending directly on knowledge of equipment deployment. In a situation where close physical proximity of equipments causes an interference condition, recourse may be made to frequency re-assignment or transmitter power variation to reduce the interference. Since this freedom may not exist in an operational system, consideration must also be given to re-deployment as a possible solution. A continuation of this discussion would lead to the observation that use of equipment deployment as an independent interference control parameter is based on "a priori" knowledge of other system parameter, which are then considered as dependent variables.

The use of geographical location as a method to reduce interference is applicable in several types of systems or environments. An example is in an environment where the equipments cannot be moved once they have been constructed or assembled in place. Another such situation is the case where a new equipment must be located in an already congested environment, and more than one location is available. A third situation in which location may be a variable is in finding an optimum placement of a number of mobile or semi-mobile equipments which are expected to remain fixed for a period of time.

Field testing of a method attempting to use geographical location as the variable by which interference reduction is achieved, requires, in addition to the parameters which must be known or measured in testing any method, the ability to relocate equipment and put it into operation easily and quickly. Also required is an area in which many different locations are available for equipment placement.

3. <u>Antenna Pattern Control</u>

Important factors in the determination of the degree of interaction between equipments are the antenna coupling parameters. When directive antennas are used, such factors as gain, side lobe and back lobe levels, polarization, direction of the main beam, and scanning modes must be considered. Since some control over these parameters is available, it is logical to attempt to utilize some of the characteristics of antennas as an operational method to reduce interference between equipments. A treatment of the problem of optimum specification of antenna parameters that attempted to be correct in every detail for realistic transmitter-receiver systems would be extremely difficult or even impossible at the present time. However, formulation of some simplified problems that are amenable to solution is both possible and useful as a first step in the direction toward developing a more comprehensive approach.

In antenna systems utilizing directive antennas, some of the methods that can be considered for effecting interference reduction are side-lobe blanking, side-lobe subtraction, sector scanning, sector blanking, and multi-beam methods. If the directive antennas are stationary (such as in certain point-to-point communication systems) beam positioning may be used to minimize interference.

As an example of this last method, consider an environment containing a number of fixed directional antennas serving communication equipments which are experiencing mutual interference. There may be an optimum direction for the main beam of each antenna which is different than the line-of-sight direction between communicating sets. Take the case, for example, in which a transmitter and receiver with separate directional antennas, closely spaced, experience interference. It may be possible to rotate each antenna so that its gain in the desired direction (that is, for communicating with its desired receiver or transmitter) is reduced, say 3 db. Then, if the rotations have been in opposite directions, the mutual gain between the two antennas may be reduced by 6 db or even more. Thus, it is possible for an increase of 3 db in the signal-to-interference ratio to be the net result of this procedure. Clearly, the application of such a method to a larger number of communication antennas results in an optimization problem.

Another type of antenna control, applicable to scanning antennas, which may be considered for interference reduction is antenna scanning control. This may take either form of sector blanking on the part of an interfering transmitting antennas, or scanning contour control, in which an antenna that normally rotates at a fixed elevation angle has a change of elevation programmed into its scanning mode. In this way it is possible to keep the main beam of the interfering antenna from illuminating the antenna of some susceptible equipment and perhaps thereby reduce the amount of interference suffered by the victim equipment. Situations can be constructed, containing several such scanning antennas, where finding the best combination of sectors to be blanked or the best scanning contours becomes an optimization problem requiring techniques of operations research for its solution.

4. Frequency Assignment

An obvious method for controlling interference in systems containing many equipments is to choose operating frequencies of the equipments in question so that interference is minimized or eliminated. One of the reasons for this is that for a large proportion of the existing and proposed equipments, tuned frequency is a parameter that is readily varied within certain well-defined limits. This is in contrast to a parameter such as power output, which is not as easily varied on many equipments. However, an attendant disadvantage of varying frequency is, that unlike power output, there is no direct relationship between frequency and a measure of interference or even to some parameter closely connected to a measure of interference, such as signal-to-noise ratio. Therefore, the techniques by which an optimum frequency assignment might be found can be considerably different than those available for making optimum power assignments.

This situation to which frequency assignment methods apply range from the extreme of a very permanent assignment, such as commercial radio and television stations, to the other extreme of a completely flexible assignment (within a band), such as exists in amateur radio operations. Many situations, especially involving military operations, fall somewhere between these two extremes of complete flexibility and rigidity. Therefore, there is a need for making frequency assignments at various intervals of time or alternatively, to have different assignments available for different well defined situations. It is assumed throughout that the frequency assignment problem deals with equipments which are by design, or perhaps by regulations, restricted to be tuned to frequencies that lie in a clearly defined band (or bands).

Finding an optimum frequency assignment for a group of equipments becomes a non-trivial problem when frequency channels must be shared among several different equipments or their required frequency bands overlap. A number of different approaches have been taken in attempting to devise frequency assignment schemes. Among them are those investigated on the previous program[1], in which one technique suggested was of the type used to solve the classical assignment problem of operations research, while other techniques developed were sub-optimum frequency assignment procedures using dynamic programming and restricted combinatorial search techniques. Similiar problems have been treated by Perlin and others[2, 3, 4, 5] in finding maximal lists of non-interfering frequencies for communications equipment by constructing mutual interference matrices. Further discussion of the optimization problem using frequency assignment as the interference control method is found below in Section V.

B. Review of Optimization Techniques

A variety of approaches is called for in research on methods of optimizing the operations of emitters and receptors within congested electromagnetic environments. One reason is that types of equipment and the factors subject to control, as well as the uncontrollable factors, will differ from situation to situation. A further important reason for multiple approaches to the general problem of optimizing the operations of electromagnetic equipment is that relatively powerful techniques may be available or discoverable which are, however, limited in scope. An example is the use of linear programming as a tool for assigning transmitter powers where the objective is to minimize the total power output summed over the group of transmitters. This elegant mathematical optimization technique is not readily applicable to variables such as frequency or location. Even if a single all-encompassing approach to electromagnetic compatibility could be devised, it would in all likelihood be computationally unmanageable.

Another reason for flexibility in dealing with the problem of optimizing interference control methods is that each particular method and the situation in which it is used requires an optimization technique specifically tailored to it. For example, an efficient optimization technique for making a frequency assignment for communication equipment spread over a large region may not be suitable for assigning frequencies to radars located in a small, congested region. This is even more obvious if a different variable is under control in each situation.

In succeeding paragraphs some mathematical optimization techniques are briefly discussed in relation to some interference problems to which they appear applicable.

1. Linear Programming. A linear programming problem is one that can be formulated as follows: find values of the variables x_j, $j = 1, \ldots, n$, such that the value of a linear function

$$z = \sum_j c_j x_j$$

is minimized (or maximized) subject to the satisfaction of a set of m linear constraints that restrict the values that the variables may assume. The i-th constraint has the form

$$\sum_j a_{ij} x_j \; R \; b_i$$

where R stands for an equality or inequality symbol ($=, <, >$). The values of the parameters a_{ij}, b_i, and c_j are specified in any particular problem.

A great deal of attention has been devoted to optimization problems of this type in recent years, from both practical and theoretical points of views. Efficient methods for obtaining solutions to linear programming problems are known, in particular the simplex algorithm originated by Dantzig, and variants thereof.

Extensions to, and special cases of, linear programming are being intensively studied. Various problems that can be viewed as particular instances of the general linear programming problem stated above are best solved by algorithms that take advantage of their special structure. Examples are provided by certain types of network optimization problems such as the "transportation problem" and, more generally, by linear programming problems to which integer solutions are desired, i. e., integer-programming problems. Progress is also being made on linear programming with uncertain information, the exact values of the parameters being replaced by probability distributions.

Relaxation of the linearity requirement leads to a further class of problems. The development of modified simplex algorithms allowing quadratic, or, more generally, convex objective functions and constraints is a large step in the direction of attacking more general classes of mathematical programming problems by methods that have proved their worth in the linear case.

2. Dynamic Programming. The theory of dynamic programming was originated in response to the need for analyzing and optimizing multistage processes, which are of more and more importance in industrial and military operations. The

ramifications of the theory have become very extensive in recent years, principally at the hands of R. Bellman, who initiated the approach, and numerous co-workers. Typically, a dynamic-programming problem is one in which a sequence of decisions must be made, earlier decisions affect the alternatives which are available later on, and there is some objective function to be maximized or minimized over the whole process. The aim of the theory is to translate the description of these problems from the concrete language of particular objects, relationships, and events into the abstract language of mathematics, and then to apply the precise and powerful techniques of that discipline. A number of novel and challenging mathematical problems have been brought to light as a result of this attempt, some of formidable difficulty. The theory has, however, been successful in providing tools for dealing with a broad range of practical problems, with particular emphasis on numerical techniques for attaining approximate solutions. The availability of automatic digital computers makes it feasible to apply these techniques on a relatively large scale.

In the course of the previous project on the application of operations research to interference, a number of mathematical models were developed leading to minimization equations of a dynamic-programming nature. The variables considered include location, frequency, and power.

By way of illustration, in one of the situations considered, there is a set of transmitters, each with its associated receivers, and the system is to be operated through successive time periods. There is also a set of frequencies available for the system, and the assignment of frequencies may be changed at the beginning of each time period. The probabilities for each transmitter and each receiver of being on during any given time period are known in advance. Suppose now that there are costs associated with changing the frequencies of the various transmitter-receiver groups, while the cost of interference during a time interval is some constant times the total interference suffered during the interval. The objective is to minimize the total expected cost through a given number of time intervals by choosing the best schedule of frequency assignments. The problem leads to a dynamic-programming-type equation that is, however, computationally difficult to solve for systems of any considerable size. There are two main reasons for this: the number of dimensions is very large; and classical analytical procedures for solving dynamic programming problems are not applicable because of the discreteness of the variables.

A number of more elementary models were also considered from the point of view of dynamic programming. For example, frequency-assignment problems were formulated for both deterministic and probabilistic conditions of operation during a single time period. Although the situations to which these models apply are not in themselves dynamic in nature, a technique often employed in solving dynamic programming problems, namely successive approximations in policy space, was suggestive. A combination of mathematical manipulation and numerical computer procedures was worked out based on this approach.

It is concluded that dynamic-programming formulation of many advanced types of interference problems is possible. Where interference problems are not essentially dynamic in nature, the theory and computational practices of dynamic programming can nevertheless, be a valuable source of ideas as to ways of solving such problems.

3. <u>Network Optimization Techniques</u>. By a network in a general sense is meant a system that can be analyzed in terms of one-to-one relationships among a set of elements.

It has been shown in work done during the past year that for one type of frequency assignment problem that can be formulated, the optimum solution can be found by efficient network algorithms, i.e., those used to solve "Classical assignment problems" of operations research. A statement of the type of frequency assignment problem in question is as follows: There are n transmitters and also n available frequencies, one frequency to be assigned to each transmitter. With respect to the n transmitters and their associated receivers, the frequencies have been found to be a non-interfering set, i.e. as far as these equipments are concerned, any assignment of the n frequencies can be made without resulting in interference. However, interference with other radio equipment in the vicinity of the transmitters can occur. Information on this potential interference is given in the form of an n by n matrix, the rows of which correspond to the n transmitters and the columns to the n frequencies. The entry in the cell of the matrix corresponding to the i-th transmitter and the j-th frequency is a measure of the interference with equipment in the vicinity of the transmitter if it is operated at that frequency. The objective is to minimize the total interference subject to the requirement that there be a complete assignment of frequencies. Formally, this is a statement of a classical assignment problem and an optimum solution can be found efficiently by employing the "Hungarian" or other algorithms for the solution of such problems.

The same mathematical model may apply to situations in which other variables are subject to control. Instead of frequencies, locations of transmitters, for example, may be assignable under a similar set of conditions.

4. Procedures for Restricted Combinatorial Search. Many of the most common yet most important interference problems do not seem to be amenable to solution by mathematical methods of special elegance and power. In any case, even if such methods remain to be discovered, it is desirable in the meantime to have a capability for dealing with such problems in a more direct fashion. The advent of the large-scale digital computer makes

it feasible to attack many problems that would otherwise defy analysis due to the sheer extent of the logical and arithmetic operations required. Nevertheless, care must still be exercised to restrict the amount of computation called for in searching for optima in interference situations, due to the fact that the number of possible system operating conditions will often far exceed the search capability of even the largest computer. In the circumstances, it may be prudent to forego the demand for an absolute optimum and settle for a suboptimum solution.

To satisfy this need for relatively straightforward and, at the same time, computationally manageable methods of searching for suboptimum solutions to interference problems, work has been done on the development of procedures for restricted combinatorial search. The approach capitalizes on the fact that in situations where there is actual or potential electromagnetic interference the individual factors subject to control are often, in a practical sense, confined to a reasonably small set of possible values. In all cases, the field of possibilities generated by combinations of values of such discrete variables is discrete. If there is an objective function to be maximized or minimized which can be evaluated for each combination, optimum combinations can, in principle, be found by a process of complete enumeration and evaluation.

Since exhaustive search is generally impossible, systematic methods for partial search merit investigation. The design and execution of restricted combinatorial searches has been considered in the context of interference, and some principles that can increase their efficiency have been stated in the Final Report of the preceding program.[1] Procedures for designing one class of restricted combinatorial searches were developed. The class of problems to which the procedures are applicable is that of searching among the permutations of a set of elements, where for every permutation there is a real number measuring its value, and the object of the search is to find a permutation with as large (or as small) a value as possible. A corresponding interference problem is the assignment of a set of frequencies to a set of transmitters, one frequency per transmitter, with minimization of the number of receivers in the environment that are interfered with.

In summary, the advantage of the general approach to optimization in interference situations that has been sketched in outline is that it is generally applicable to any controllable factor (frequency, location, type of equipment, etc.) which can be described in terms of discrete alternatives, and to any chosen objective function; the disadvantage is that an absolute optimum may not be found due to excessive computational requirements.

C. Validating and Testing Interference Control Methods

An interference problem in a theoretical system of emitters and receptors can be solved by using one or more of the control methods discussed in conjunction with an efficient optimization technique, which determines the optimum theoretical solution. The correspondence between the theoretical solution arrived at in this manner and the actual (real) solution will depend upon the validity of the assumptions made in mathematically describing the equipments used, the propagation characteristics and the criteria to determine when and to what degree interference occurs. An interference test in this context might then be interpreted as a measure of the correspondence between the theoretical and actual solution of a given compatibility problem, and the degree of correspondence observed would infer the relative validity of the theoretical solution.

Since the criteria to determine when and to what extent interference occurs are directly related to system performance, an interference test of the theoretical solution implies the measurement of actual system performance under experimentally controlled conditions. The conditions are derived from theoretical analysis and solution of the system compatibility problem. This analysis requires as input data actual equipment characteristics. To the extent to which these are not accurately known, the initial test procedures are required to provide some of them. Thus, the experimental conditions are frequently modified during the course of a test by the increased knowledge of the equipment and system performance gained during the test itself. Ideally, after sufficient data on equipment performance has been collected by means such as the Department of Defense Compatibility Program, this initial phase of a test program as planned on this project will be reduced to a minimum.

After the required data on actual system performance has been gathered, comparison of the predicted and actual system performance will yield the degree of correspondence. The predicted system performance may be interpreted as the minimum achievable interference for the given situation and the control method employed, while the actual system performance indicates how nearly the theoretical solution can be achieved.

There are three main sources of error which prevent the correspondence between the predicted and actual performance from being perfect. One involves the error resulting from the assumptions made in deriving the theoretical solution. Another is introduced by the errors inherent in the spectrum signature data which must be used in the analysis. Since measurement of equipment spectrum signatures is not within the scope of this program, these data will be obtained from the literature and from other organizations currently collecting such data, with only sample measurements made of some critical characteristics to minimize this source of error. The third set of errors involved concerns the measurement accuracy of the data taken during these tests. The most obvious source of error here results from the fact that the field test usually does not represent a 100 percent controlled experiment, and the lack of control over all electromagnetic radiators in the vicinity of the test site introduces extraneous and fluctuating signals which may not always be distinguishable from the test signals. The subsequent comparison and analysis of theoretical and actual system performance should shed some light on the relative seriousness and overall magnitude of these errors.

IV. SELECTION OF CONTROL METHOD

The criteria used to select a particular interference control method for further study and validation by field testing were primarily (1) the potential usefulness of the method, (2) the applicability of techniques of operations research to the attendant optimization problem and the discovery of efficient mathematical techniques to fit the particular interference situation being treated, and (3) the feasibility of submitting the method to a field test using the facilities of the Rome Air Development Center.

Based on these considerations, the method selected to receive primary attention in the ensuing investigations and test program on the project is frequency assignment.

Of the system parameters subject to operational control, the most widely available is the tuned frequency of the equipments comprising the system. Frequency control therefore, fulfills one of the requirements of a useful interference reduction method. The other requirement is, of course, that variation of the parameter in question produces significant changes in the interference experienced by the system. Clearly, frequency changes do have this effect. It was furthermore considered desirable to employ this method with radar, since considerable work has already been done by others in applying the method to communication equipment.

The applicability of a number of techniques of operations research to the problem of finding an optimum frequency assignment is already clear from the discussion in the previous sections and the work accomplished in this area on the preceding project. Additional effort to tailor various optimization techniques to the method of frequency assignment has been expanded and is reported in Section V below.

Finally, the feasibility of submitting this method to a field test was considered. It was determined, after study of the facilities available at the Verona Test Site and after making some preliminary calculations such as those described below, that a meaningful field test of the validity of frequency assignment as an interference control method for radars was feasible, utilizing the Verona Site. It is believed that a similar test using communications equipment available at the Rome Air Development Center for this purpose would be inherently more difficult. Among the reasons for this is that the signal spectra and receiver bandwidths of radars are generally much wider (compared to their tuning ranges) than those of communication equipments, the transmitter power outputs of radars are much greater and radar receivers are often somewhat more sensitive than communications receivers. These factors all contribute to creating greater interference problems for radars than for communications equipment. Therefore, since only a limited number of either type of equipment is available for test purposes, it was concluded that with frequency as the control variable, a field test using radar would be more feasible than one conducted with communications equipment.

V. SOME INTERFERENCE MINIMIZATION PROBLEMS WITH TUNED FREQUENCY AS THE CONTROLLABLE FACTOR

A. Introduction

Tuned frequency of transmitters and receivers of electromagnetic radiation is the operational variable most widely available to users of such equipment for the purpose of attempting to achieve compatibility in congested environments. In recognition of this fact, a considerable amount of effort has been devoted to the problem of developing effective procedures for the assignment or selection of operating frequencies. Because of the fundamental importance of frequency as a controllable variable, it seems worthwhile to re-examine some of the typical situations in which there is some degree of freedom of choice with respect to this factor, and to formulate, within the framework of mathemmatical programming, various optimization problems to which these situations give rise.

The reason for approaching these problems from the point of view of mathematical programming is that in some cases it may be possible to apply relatively efficient known procedures for finding optimum solutions; in other cases, it may be possible to develop efficient solution procedures on the basis of general mathematical programming principles.

B. Formulation of Interference Problems Involving Choice of Frequency

A broad class of frequency assignment problems may be characterized in general terms as follows.

Given are a set $\{R\}$ of receivers indexed by $i = 1, \ldots N_I$ and a set $\{T\}$ of transmitters indexed by $j = 1, \ldots, N_J$. The set of receiver indices, $\{1, \ldots, N_I\}$, will be denoted by $\{I\}$ and the set of transmitter indices, $\{1, \ldots, N_J\}$, by $\{J\}$.

A variable D_{ij} which can take on only boolean values, i. e., 0 and 1, will serve to distinguish desired from undesired transmissions. Let $D_{ij} = 1$ if it is desired that receiver R_i accept output from transmitter T_j, and let $D_{ij} = 0$ otherwise.

Also given is a discrete set of frequencies $\{F\}$ indexed by $k = 1, \ldots, N_K$ that are available for the operation of the transmitters and receivers in question. The set of frequency indices, $\{1, \ldots, N_K\}$, will be denoted by $\{K\}$. Included in $\{F\}$ are all the fundamental frequencies to which it is considered permissible to tune any of the equipments over which control can be exercised. Where the tuning range of a piece of equipment may actually be continuous, a discrete set of tuning points is chosen. Each tuned frequency will be surrounded by a frequency band of the electromagnetic spectrum and there may be significant emission or response at harmonic and spurious frequencies as given by the spectrum signature.

The set of frequencies to which receiver R_i can be tuned will be denoted by $\{F_i\}$ and the set of frequencies to which transmitter T_j can be tuned will be denoted by $\{F_j\}$. The corresponding sets of frequency indices will be denoted by $\{K_i\}$ and $\{K_j\}$. $\{F_i\}$ and $\{F_j\}$ are subsets of $\{F\}$ and likewise $\{K_i\}$ and $\{K_j\}$ are subsets of $\{K\}$. The number of indices in $\{K_i\}$ and $\{K_j\}$ will be denoted by N_{K_i} and N_{K_j} respectively. For $D_i = 1$, $\{F_i\}$ will usually be identical to $\{F_j\}$.

It will be convenient to utilize index variables for the frequencies to which individual transmitters and receivers may be tuned. The frequency index variable k_i, for receiver R_i, will range only over $\{K_i\}$, and the frequency index variable k_j, for transmitter T_j, will range only over $\{K_j\}$. The frequency corresponding to particular value of k_i or k_j will be denoted by F_{k_i} or F_{k_j} as the case may be. If $D_{ij} = 1$, only values of k_i and k_j such that $k_i = k_j$ need be considered.

The effect on receiver R_i of the output of transmitter T_j when the former is tuned by frequency k_i and the latter to frequency k_j will be denoted by $A_{k_i k_j}$. This effect will depend upon the types of equipment, modes of operation, locations, atmospheric conditions, etc. Measurement and predicticn procedures are assumed capable of providing reasonably accurate information concerning the effects of transmitters on receivers. In various circumstances $A_{i_k k_j}$ might be a boolean number, an integer, a rational number, or a vector.

Table I is a skeleton format for the information that has been specified. Receiver indices are given on the left, and transmitter indices along the top of the table. Frequency indices are subsumed under both receiver and transmitter indices in accordance with the fact that the response of a receiver to a transmitter is a function of the tuning of both. The matrix of values of $A_{k_i k_j}$ making up the body of the table is thus composed of submatrices, one submatrix for each receiver-transmitter combination.

The distinction between desired and undesired transmissions is made in Table I by enclosing submatrices for which $D_{ij} = 1$ in heavy lines and by blacking out the off-diagonal portions of these submatrices since, for a desired transmission, the receiver and transmitter will be tuned to the same frequency.

A "dummy" transmitter with index 0 has been included in Table I to account for receiver inputs other than those due to the transmitters specifically listed. The interference effects of transmitters other than those over which control can be exercised, together with background noise originating both external and internal to the receiver, can be combined in this category. Receiver effects attributable to the dummy transmitter can vary from one tuned frequency to another.

In Table I four receivers, four transmitters, and four frequencies are indicated for illustrative purposes, with desired communication between those transmitters and receivers having the same index number. The variety of particular situations to which the general scheme applies is very wide as indicated by the following comments.

Various types of transmitters and also various types of receivers, including both communication and radar equipment, can be included in a single table. In the case of radar the desired transmitter for a given receiver will generally be co-located.

Associated with different pieces of equipment may be identical, partially overlapping, or disjoint subsets of the complete set of tuned frequencies, $\{F\}$.

Any number of receivers may desire a single transmitter. Co-channel operation of transmitters may be permissible.

Let x_{jk} be a boolean variable relating frequencies to transmitters. If frequency k is assigned to transmitter j than $x_{jk} = 1$; otherwise $x_{jk} = 0$. The general optimization problem of concern is to find assignments of frequencies to transmitters (and their associated receivers), i.e., values of x_{jk}, such that certain specified receiver performance criteria and other constraints are satisfied and at the same time some overall measure of effectiveness is maximized or minimized.

TABLE I

FORMAT FOR DATA ON INTERFERENCE AS A FUNCTION OF ASSIGNED FREQUENCY IN A COMPLEX OF TRANSMITTERS AND RECEIVERS

RECEIVER	TRANSMITTER	0	1				2				3				4			
	TUNED FREQUENCY	—	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	1																	
	2																	
	3																	
	4																	
2	1																	
	2																	
	3																	
	4																	
3	1																	
	2																	
	3																	
	4																	
4	1																	
	2																	
	3																	
	4																	

C. A Problem of Optimum Frequency Assignment Where Receiver Inputs Have an All-or-None Character

One specfic model of frequency assignment that appears particularly applicable to many radar interference situations is as follows. Let the entries $A_{k_i k_j}$ in the body of a frequency-interference table of the sort that has been described be restricted to the boolean values 0 and 1. $A_{k_i k_j} = 1$ has the interpretation that when receiver R_i is tuned to frequency k_i and transmitter T_j is tuned to frequency k_j there is significant input to the receiver from the transmitter; $A_{k_i k_j} = 0$ has the interpretation that there is no significant input. The coupling between transmitters and receivers is thus assumed to have an all-or-none character. If $D_{ij} = 1$ this input constitutes the desired signal; but if $D_{ij} = 0$ it constitutes interference or potential interference. One reasonable aim is to assign frequencies to transmitters in such a way that the maximum possible number of interference-free transmissions is realized.

Using the notation previously developed this problem can be stated thus. Given a frequency-interference table with

$$D_{ij} \in \{0, 1\}$$

$$\sum_j D_{ij} = 1 \qquad i \in \{I\}$$

and with entries

$$A_{k_i k_j} \in \{0, 1\}$$

find values

$$x_{jk} \in \{0, 1\}$$

such that

$$\sum_{\substack{i,\ j,\ k \\ k_i = k_j}} A_{k_i k_j} \ D_{ij} \ x_{jk}$$

is a maximum subject to the constraints

$$\sum_k x_{jk} \leq 1, \quad j \in \{J\}$$

and

$$\sum_{\substack{i,\ j,\ k \\ k_i \in \{k_i^*\} \\ k_j \in \{k_j^*\}}} A_{k_i k_j} \ (1-D_{ij}) \ x_{jk} = 0$$

An asterisk is placed on each entry $A_{k_i k_j}$ satisfying the three conditions:

(1) $D_{ij} = 1$; (2) $x_{jk} = 1$; (3) $k_i = k_j$. Then $\{k_i^*\}$ is defined as the subset of rows of the table containing an asterisk and $\{k_j^*\}$ is the subset of columns of the table containing an asterisk.

An example is given in Table II. Submatrices for which $D_{ij} = 1$ are enclosed by heavy lines. In this example there are altogether five transmitters, six receivers, and six frequencies. The desired transmitter for receivers No. 1 and No. 6 is No. 1, and the permissible frequencies are Nos. 1 through 5. Transmitter No. 2 is desired by receiver No. 2 with permissible frequencies Nos. 1 through 4, and so on.

The asterisks in Table II designate a frequency assignment under which all desired transmissions can occur without interference. That this is so is shown by the fact that at the intersection of every row containing an asterisk with every column containing some other asterisk the value of $A_{k_i k_j}$ is 0. These crucial zeros are typed in; elsewhere in the table zeros are indicated by blanks.

The type of problem described is in some respects similar to the problem of finding maximal lists of non-interfering frequencies in communication systems as studied by Perlin and others.[2, 3, 4, 5] Methods such as have been developed for finding solutions to the latter problem are in principle applicable here.

D. Classical Assignment Problem for Frequencies

A frequency assignment problem that can be treated as a classical assignment problem of operations research has been described in Section III. B. 3., and more fully in (1). Efficient algorithms (e. g., the Hungarian algorithm) for finding

Table II

Example of Interference-Free Frequency Assignment Where Coupling of Transmitters and Receivers Has an All-or-None Character

	j	1					2				3				4			5					
i	k	1	2	3	4	5	1	2	3	4	2	3	4	5	4	5	6	1	2	3	4	5	6
1	1	1	-	-	-	-	1	1	1	1								1	1				
	2	-	1	-	-	-	1	1	1		1					1			1	1	1		
	3	-	-	1*	-	-	1	0	1			1		0	0				1	1	1	1	0
	4	-	-	-	1	-		1	1	1			1		1			1	1		1		
	5	-	-	-	-	1		1			1			1		1	1				1	1	
2	1	1	1	1			1	-	-	-	1		1					1					1
	2	1	1	0			-	1*	-	-	1		1	0	0				1				0
	3	1	1	1	1		-	-	1	-		1						1	1	1			
	4			1	1		-	-	-	1	1	1	1		1			1		1	1		
3	2	1	1					1		1	1	-	-	-		1			1		1		
	3			1	1				1		-	1	-	-		1				1			
	4		1		1	1		1		1	-	-	1	-							1		1
	5			0		1	1	0			-	-	-	1*	0						1	1	0
4	4	1		0	1			0		1			1	0	1*	-	-				1	1	0
	5		1			1								1	-	1	-				1	1	
	6			1					1						-	-	1		1			1	1
5	1	1	1				1	1		1						1		1	-	-	-	-	-
	2		1				1	1	1	1	1				1			-	1	-	-	-	-
	3		1	1	1		1	1	1			1	1					-	-	1	-	-	-
	4		1		1	1		1	1	1			1		1			-	-	-	1	-	-
	5		1		1	1			1			1		1		1		-	-	-	-	1	-
	6			0				0			1	1		0	0		1	-	-	-	-	-	1*
6	1	1	-	-	-	-	1			1								1		1	1	1	
	2	-	1	-	-	-		1	1	1	1								1	1		1	1
	3	-	-	1*	-	-	1	0	1		1	1		0	0			1	1	1	1		0
	4	-	-	-	1	-		1	1	1	1	1	1		1				1	1	1	1	
	5	-	-	-	-	1		1						1	1	1				1		1	1

optimum solutions to such problems are known. In terms of the scheme shown in Table I a problem of this sort arises where effects of assigning frequencies to transmitters can be considered independently of the receivers which have these as their transmitters. In other words, entries in the body of the frequency-interference table are reduced from $A_{k_i k_j}$ to A_{k_j}.

VI. FIELD TEST

A. Equipment Selection

After the control method to be investigated was chosen to be frequency assignment, and it was decided to utilize the radars available at the Verona Test Site, a study was made of these radars, their characteristics and suitability for the type of test contemplated. The information used came primarily from the RADC report on the Verona ECM Engineering Test Facility,[6] from data supplied on request by the sponsoring agency, from reports on radar measurement programs,[7,8] and from radar technical manuals. Based on this information it was concluded that the initial tests should be conducted with the L-band radars at Verona, with possible expansion to radars in other bands as the test proceeds.

Given that a relatively small number of equipments is to be used in the test, and that frequency is the controlled variable, it is virtually imperative that all the radars operate in the same band in order to provide a potential interference environment which can be considered to simulate a large congested environment. The L-band radar facilities at Verona meet this requirement fairly well.

B. Sample Interference Calculations for Closely Spaced Radars

In order to confirm the estimate that a meaningful interference test can be conducted at the Verona site, a few preliminary calculations were made for a situation similar to that expected on the site. It is first necessary to check that the received signal powers at a given receiver as a function of frequency are variable over a wide enough range so that a change in the tuned frequencies of the equipments does actually produce a change in the amount of interference experienced by the receivers in the system. The received signal power is also, of course, dependent on the distance between radars and their antenna orientations, so that these factors must also be taken into account, and may be used to help provide the desired conditions for the field test. A second consideration is to determine the existence of spurious receiver responses and spurious transmitter emissions. Since the initial measurements will be made with radars operating in the same band, the former assumes a greater importance than the latter, since more spurious responses are likely to be found in the tuning band than spurious emissions.

1. Received Power Calculations

Consider first two radars separated by some specified distance. Assume that propagation loss is that of free space and the two radars have the following characteristics:

	Radar A	Radar B
Peak Power Output	400 kw	2000 kw
Antenna Gain	560	3200
Frequency	1300 mc	1300 mc

The receiver signal power is given by

$$P_r = \frac{P_t G_r G_t \lambda^2}{(4 \pi R)^2}$$

where

P_r = received power

P_t = transmitted power

G_r = receiver antenna gain

G_t = transmitter antenna gain

λ = wavelength

R = distance between antennas

Case 1: Consider the separation R to be 600 feet or 168 meters.

Using the formula and data given above, and assuming the transmitter to be Radar B with the main beam of its antenna pointing at the receiving antenna, Radar A, the received power is

$$P_r = 36.0 \text{ kw or } 75.6 \text{ dbm}$$

Case 2: Consider the separation R to be 2000 feet or 610 meters

Keeping all factors except the separation fixed, the received power is

$$P_r = 3.24 \text{ kw or } 65.1 \text{ dbm}$$

To take the receiver antenna response pattern into account, assume that its main lobe is 18°, the side lobes are 30 db below the main lobe and occupy a total of 90°, and that the back lobe is 40 db below the main beam, occupying the remaining 252°. Thus, if the receiving antenna is rotating, the fixed transmitting antenna "sees" the receivers main lobe 5 percent of the time, the side lobes a total of 25 percent of the time and the back lobe 70 percent of the time. Therefore, the received power is 36,000 watts 5 percent of the time, 36.0 watts 25 percent of the time and 3.6 watts the 70 percent of the time for Case 1. In Case 2 the received powers are respectively 3240, 3.24 and .324 watts.

If now the two radars are no longer tuned to the same frequencies, the attenuation due to the receiver selectivity becomes a factor. Data available for typical search radars indicates that the receiver response may be as much as 100 db down from the tuned frequency response for signals on the order of 10 mc removed from the tuned frequency. Thus, for some frequencies in the band, the received signal powers listed above must be multiplied by a factor of 10^{-10} to give the actual receiver response. It is clear, therefore, that simply a change in frequency for either the receiver or transmitter can cause the difference between very heavy interference and virtually no interference. It must be remembered, however, that this is a very oversimplified view of the situation, which is seriously modified when receiver spurious responses are taken into account. It is for this reason that a non-trivial optimization problem exists in making a frequency assignment for several radars. If there were no spurious responses (and also no in-band spurious emissions) the simple rule that radars must be separated in frequency by more than some set amount, say 10 mc, would suffice. Unfortunately, the problem is not so simple. However, it appears from the above considerations that frequency assignments may be found which differ significantly in the amounts of interference in the system.

2. Spurious Response Calculations

A series of calculations were performed to find the spurious responses of the radar receivers for which sufficient information was available. To find these frequencies, the classical formula for receiver spurious responses was used:

$$f_{sp} = \frac{p f_{l.o.} \pm f_{i.f.}}{q}$$

where

f_{sp} = the spurious response frequency

$f_{l.o.}$ = the frequency of the receiver local oscillator

$f_{i.f.}$ = the intermediate frequency of the receiver

p, q = are positive integers

For purposes of these field tests, the spurious responses of interest are those in and close around L-band, or roughly in the tuning range of the L-band radars plus an i.f. frequency above and below. Calculations show that spurious responses fall into this relatively narrow band only when $p = q$. The relative position of these frequencies and spurious responses can be more easily seen in Figure 1. Consider the case of a radar with the local oscillator tuned above the tuned frequency of the receiver.

It is seen from the figure that the higher order responses converge in frequency toward the local oscillator frequency. Also it is expected that the magnitude of the response will decrease with increasing order, as shown in Figure 1. However, many of these higher order responses will not be significant or even observed in the field.

C. Field Test Plan

i. General

The purpose of the field test is to provide a check on frequency assignment as a control method for the application of operations research techniques to radar interference reduction. The field test will also provide field measurement data not already available from measurements made on other programs on these radars, for input to an interference prediction computer program.

The essential portion of the test, the interference

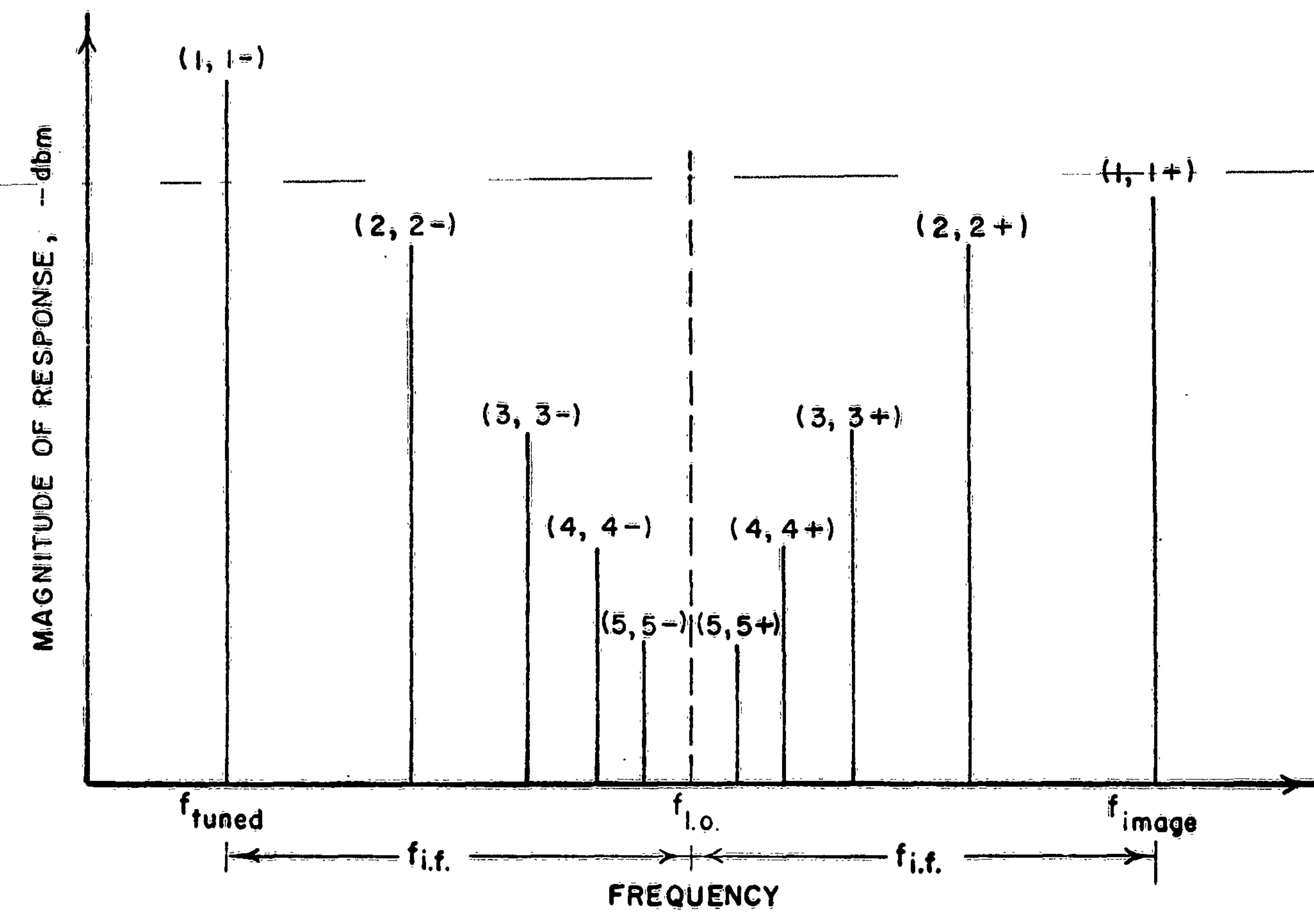

FIG. I RECEIVER SPURIOUS RESPONSES WITH $p = q$

measurements, will consist of three different determinations of the amount of interference present. They are described more fully below; however, briefly, one is a subjective determination by an operator, the second is the proportion of the PPI scope which displays interference pulses, while the third is an actual count of interference pulses above a given threshold at the radar output. Interference at each radar receiver will be measured for a variety of configurations, including one or more interfering transmitters operating, several combinations of antenna orientation or scanning modes, and, of course, a number of different frequency assignments.

It should be pointed out that the second part of the tests proposed here, consisting of the actual interference measurements, is somewhat provisional in nature, since the best procedures to follow will depend to a certain extent on the outcome of the general measurements to be made during the first portion of the test. Therefore, it is quite probable that modifications in the frequency assignments and specific procedures will be made during the course of the testing.

The field test is to be conducted using three L-band radars available at the site. These radars will be referred to as transmitters T_1, T_2, and T_3, and as receivers R_1, R_2, and R_3.

The measurements of this field test will be divided into two categories: general measurements, to include spurious emissions and responses of the radars, coupling factor and sensitivity measurements; and interference measurements.

2. General Measurement Procedure

(a) Transmitter spurious emission - Using a field intensity meter at about 2000 ft. from the transmitting antenna and in the main lobe, the relative field intensity of emission of the transmitter will be measured, tuning the FIM from f_1 to f_2 while the transmitting radar is tuned to f_3. The height of the FIM antenna must be adjusted until the received signal is approximately maximum. These measurements will be made according to the following table:

TABLE III

Field Intensity Measurement - Frequency Ranges

Transmitter	f_1	f_2	f_3
T_1^*	1220	1410	1290 mc 1315 1340
T_2^*	1220	1410	1260 mc 1300 1340
T_3^*	1220	1410	1260 mc 1285 1340

* Antenna not rotating.

The measurements are to be made in 10-11 steps of frequency from f_1 to f_2, the exact frequencies determined by: (1) significant points in the spectrum, such as maximum indications on the FIM (indicate the data as such); or (2) uniform steps of frequency from f_1 to f_2 if no such significant points of (1) are found.

The fundamental peak power output of the radar will be measured and the spurious emissions recorded in db below the power at the fundamental frequency.

(b) Receiver Spurious Responses and Sensitivity - Using a standard L-band generator fed into the input to the receiver, the spurious responses of each of the radar receivers is to be measured over the band f_1 to f_2 while the receiver is tuned to f_3, as shown in the following table:

TABLE IV

Spurious Response Measurement - Frequency Ranges

Receiver	f_1	f_2	f_3
R_1	1220	1410	1290 mc 1310 1340
R_2	1220	1410	1280 mc 1310 1340
R_3	1160	1470	1280 mc 1310 1340

The actual frequencies of measurement are to be determined at the time of the field test by (1) the points in the band with peak response, or (2) if no such discrete points are apparent, a set of uniformly spaced points from f_1 to f_2.

In the cases of the receivers with AGC, MTI, FTC, or other specialized features that affect the selectivity and sensitivity of the receiver, these features should be shut off or otherwise disabled for this test, wherever practicable. The condition of these features ("on" or "off") should be noted when taking the data.

At each of the assigned tuned frequencies of the receivers the minimum detectable signal (MDS) is to be determined by using more or less standard methods of inserting a pulse signal (preferably synchronized with the p. r. f.) into the receiver input and slowly raising the pulse generator power until the generated pulse is just detectable over the grass of the A scope, and recording the generator output level.

(c) Coupling Factor - The coupling factor is to be measured between each of the radars as a transmitter and the other two as receivers. The measurement is to be made by the substitution method with the respective antennas pointing directly at each other. All the radars and their receivers are to be tuned to 1320 mc. Also, when a measurement is made at a radar receiver, that radar is <u>not</u> to be transmitting simultaneously.

The input power of each receiver is to be measured by a substitution technique. With the gain of the receiver adjusted to display the received pulses on the A-scope or a test scope with considerable deflection, but below limiting, the height of the received pulses is noted. In order to achieve such a display of the received pulses, it will most likely be necessary to insert known amounts of attenuation between the antenna and receiver. Then, disconnecting the receiver from the antenna and attenuator, and inserting a pulse from a standard pulse generator, the level of the pulse generator is adjusted until the same deflection is again obtained on the scope, and the power output of the pulse generator is recorded.

With the radars as close as they are to each other, a considerable amount of attenuation may have to be added between the receiving antenna and the receiver. This attenuation in db is added to the output power of the generator to obtain the received pulse power.

3. General Measurements - Data and Results

(a) Transmitter Spurious Emission - The spurious emissions of each of the radar transmitters was measured with a Polaroid FIM meter with its L-band antenna as specified in the procedure, at a distance of 2100 feet. The data of these measurements is shown in Table V. The power density measurements were taken for the most part at significant peaks in the output spectrum.

TABLE V

Transmitter Spurious Emissions

Tx Freq.	Tx Power Peak	FIM Freq. (mc)	FIM Input Att'n db	FIM Meter Reading db/μ v	Cable Loss db	Horn Gain db	Power Density dbm
Transmitter T_1							
1290	**1 Mw**	1290	60	55	4.5	10.3	+ 2.2
		1220	20	47	4.5	10	-45.25
		1250	20	47	4.5	9.75	-46.5
		1331	20	52	5.6	10.6	-40.6
		1385	20	52	6.7	10.9	-38.2
1315	**1 Mw**	1315	60	54	5.3	10.5	+ 1.8
		1241	20	46	4.5	10	-46.5
		1271	20	52	4.5	10.2	-40.7
		1374	20	54	6.5	10.9	-37.4
		1385	20	51	6.7	10.9	-40.2
1340	**1 Mw**	1340	60	54	5.8	10.6	+ 2.2
		1241	20	47	4.5	9.9	-45.4
		1271	20	51	4.5	10.2	-41.7
		1390	20	45	5.2	10.5	-46.3
		1381	20	48	6.6	10.8	-43.2

TABLE V (Cont'd)

Transmitter Spurious Emissions

Tx Freq.	Tx Power Peak	FIM Freq. (mc)	FIM Input Att'n db	FIM Meter Reading db/μv	Cable Loss db	Horn Gain db	Power Density dbm
Transmitter T_2							
1260 mc	2 Mw	1220	20	52	4.5 db	9.75db	-40.25
		1240	20	58	4.5	10	-35.5
		1260	80	54	4.5	10	+21.5
		1300	20	53	5.0	10.5	-39.5
		1320	20	50	5.2	10.5	-42.3
		1350	20	42	6.0	10.75	-49.75
		1390	20	40	6.8	10.9	-51.1
1300	2 Mw	1255	20	44	4.5	10	-48.5
		1300	80	45	5.0	10.5	+12.5
		1322	20	47	5.4	10.5	-45.1
		1365	20	37	6.3	10.9	-54.6
		1390	0	57	6.8	10.9	-54.2
1340	2 Mw	1248	0	50	4.5	10	-62.5
		1290	0	55	4.5	10.3	-57.8
		1340	80	53	5.8	10.6	+21.2
		1394	0	52	6.9	10.9	-59

TABLE V (Cont'd)

Transmitter Spurious Emissions

Tx Freq.	Tx Power Peak	FIM Freq. (mc)	FIM Input Att'n db	FIM Meter Reading db/μv	FIM Cable Loss db	Horn Gain db	Power Density dbm
Transmitter T_3							
1260	400 kw	1260	40	53	4.5	10.1	-19.6
		1220	0	32	4.5	9.8	-80.3
		1272	0	46	4.5	10.2	-66.7
1285	400 kw	1285	40	38	4.5	10.3	-34.8
		1250	0	42	4.5	10.0	-70.5
1340	400 kw	1245	0	36	4.5	9.9	-84.4
		1274	0	34	4.5	10.2	-78.7
		1296	0	45	4.5	10.3	-67.8
		1340	40	53	5.8	10.7	-18.9

Horn used - Model CA-L Polarad Horn Antenna
Cable used - 30 ft. RG9B/U

Horn gain values obtained from calibration graph
Cable loss figures were obtained from graph calculated by measurement team at Verona Test Site.

For each measurement, the power density was calculated by the following expression:

$$\underset{(db)}{\text{Input Att'n.}} + \underset{(db/\ v)}{\text{Meter Reading}} + \underset{(db)}{\text{Cable Loss}} - \underset{(db)}{\text{Horn Gain}} - \underset{(db)}{107} = \underset{(dbm)}{\text{Power Density}}$$

The 107 db figure is the subtractive conversion from voltage in decibels above one microvolt to power in decibels above one milliwatt, referenced to 50 ohms, the nominal input impedance of the receiver used.

(b) Receiver Spurious Responses and Sensitivity - The measurements of the tuned and spurious responses for each of the receivers were made. The data of these measurements is tabulated in Table VI. Indication of the response is given in the MDS (minimum detectable signal) column, since the measurement procedure for each frequency is the standard MDS procedure. In each case, the generator frequency is adjusted to given maximum response.

For each measurement, the response is the power setting of the signal generator in decibels above one milliwatt, and the Input Attenuation is that attenuation between the signal generator and the receiver input. The algebraic sum of these two is the MDS of the receiver at that frequency.

(c) Coupling Factor - The coupling factor measurements were completed and this data along with the calculations are shown on Table VII. Difficulty was encountered in making the coupling factor measurement by the substitution method specified, due to the high power present at the receiver. The received power was then measured using the Polarad FIM as a voltmeter at the receiver input or at the directional coupler of the wave-guide, and this voltage converted to power in decibels above one milliwatt for a 50 ohm load.

TABLE VI

Spurious Response Measurements

Rx Freq.	(p, q, ±)	Gen. Freq.	MDS	Response	Input Att.	Remarks*
			Receiver R_1			
1340	(1, 1, -)	1340	-110dbm	-65dbm	45db	1329-1356
	(1, 1, +)	1400	-81dbm	-36 dbm	45db	1396-1404
1310	(1, 1, -)	1310	-110dbm	-65dbm	45db	1299-1325
	(1, 1, +)	1370	- 81dbm	-36dbm	45db	1367-1374
1290	(1, 1, -)	1290	-109dbm	-64dbm	45db	1282-1311
	(1, 1, +)	1350	-78dbm	-33dbm	45db	1348-1354
			Receiver R_2			
1280	(1, 1, -)	1280	-111dbm	-69dbm	42db	1270-1292
	(1, 1, +)	1340	-103dbm	-61dbm	42db	1331-1348 3:1 signal noise ratio
1310	(1, 1, +)	1310	-110dbm	-68dbm	42db	1300-1321
	(1, 1, -)	1250	-99dbm	-57dbm	42db	1242-1258
1340	(1, 1, +)	1340	-111dbm	-69dbm	42db	1331-1356
	(1, 1, -)	1280	-98dbm	-56dbm	42db	1275-1290
			Receiver R_3			
1340	(1, 1, +)	1340	-102dbm	--	--	
	(1, 1, -)	1222	-40dbm	--	--	3:1 ratio

TABLE VI (Cont'd)

Spurious Response Measurements

Rx Freq.	(p, q, ±)	Gen. Freq.	MDS	Response	Input Att.	Remarks
1280	(1, 1, -)	1280	-104dbm	--	--	3:1 ratio
	(1, 1, +)	1395	-40dbm	--	--	3:1 ratio
1310	(1, 1, -)	1310	-103dbm	--	--	3:1 ratio
	(2, 2, +)	1280	-17dbm	--	--	3:1 ratio
	(1, 1, +)	1430	-38dbm	--	--	3:1 ratio

*Notes:

The frequency range shown in the Remarks column is the range over which the signal was above the noise when the signal input of Receiver R_1 was -45 dbm and for the Receiver R_2 was -42 dbm.

The 3:1 ratio indicated is the criterion for standardizing the r.f. gain of each of the receivers. The procedure is to adjust the gain of the receiver so that a signal pulse just below limiting on the A-scope will be three units high while level of the grass is one unit.

TABLE VII

Coupling Factors

Transmitter No.	Transmitter Peak Power dbm	Receiver No.	Received Power - FIM Attenuation db	Received Power - FIM Meter Reading db/μv	Received Power - FIM Power dbm	Coupling Factor db
T_1	90.1	R_2	81	46	+21	69.1
T_2	86.7	R_1	80	47	20	66.7
T_1	89.2	R_3	70	53	16	73.2
T_3	88.2	R_1	70	52	15	73.2
T_2	86.7	R_3	70	50	13	73.7
T_3	88.2	R_2	80	38	11	77.2

Received Power (dbm) = Attenuation (db) + Meter Reading (db/μv) - 107 (db)

Coupling Factor (db) = Transmitted Peak Power (dbm) - Received Peak Power (dbm)

The coupling factor in this case is the ratio of peak transmitted to received power, or the difference of the powers expressed in decibels. These calculations are shown in the table. The transmitter power shown is the peak power, found by dividing the average power by the duty cycle of the radar.

The general measurements data obtained in the course of the field test will be used primarily to make interference predictions. Finding optimum frequency assignments for a configuration of radars requires that an evaluation be made of the interference existing for any given assignment, these predictions then used in the type of optimization technique discussed in Section V. The input data required for the prediction program to be used for this task are such transmitter and receiver characterististics as the peak power output, pulse width, antenna main beam gain and width, pulse repetition frequency, operating frequency, the overall receiver selectivity characteristics, spurious transmitter emission frequencies and levels, spurious receiver response frequencies and levels, and equipment locations. Whenever they are available, measured data will of course be used, either the measurements made during the test or those reported in the literature. In all other cases the nominal characteristics for the radars will be used.

4. <u>Interference Measurements</u>

For each instance of measuring the amount of interference, three methods will be used:

(a) The degree of interference at the PPI scope of the receiving radar is to be noted in accordance with the following classification:

(1) No interference - no visible interference

(2) Light interference - visible interference which, however, would not seriously hinder the operator in carrying out his mission.

(3) Medium interference - visible interference which would seriously impede the operator in carrying out his mission without making it impossible to do so.

(4) Heavy interference - sufficient interference to make it impossible for the operator to carry out his mission.

(b) If the amount of interference noted in (1) above is other than none at all, a photograph of the PPI display, or an estimate of the total degrees of the sectors of interference on the PPI is to be made.

(c) Make a count of the number of interference pulses at the video output, using the setup shown in Figure 2.

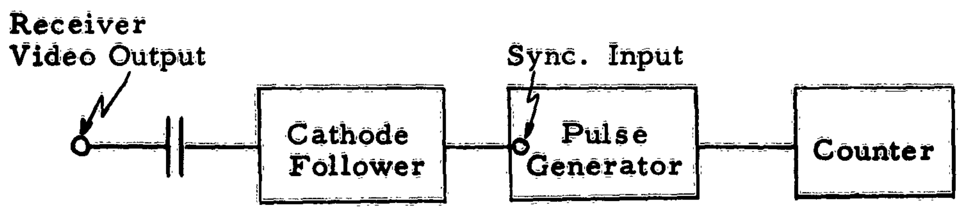

FIG. 2 PULSE COUNTING SETUP

The triggering level of the pulse generator sync circuit must be variable for adjusting the false alarm rate. A generator such as the Electro-Pulse, Inc. Model 3450 D is adequate.

If a video jack is not available, then it is sufficient to take the signal from the plate of the final video amplifier, with all ECM circuits turned off. The trigger level of the pulse generator is to be adjusted, when the interfering radar is not transmitting, to yield a false alarm rate of approximately 10 false alarm pulses per 10 sec. period. The width of the generator output pulse is to be the same as that of the interfering radar. With the pulse generator so adjusted, count the number of pulses received from the interfering radar over five 10 sec. periods. Readjust the false alarm rate to 100 pulses per 10 second time base, and again count the number of interfering pulses over five 10 sec. periods.

At the same time that the interference pulse count is taken at a receiver, determine the actual p. r. f. of the interfering radar.

These interference tests will be made for each of the following frequency assignments:

TABLE VIII

Frequency Assignments

Assignment	T_1	T_2	T_3
a.	1310 mc	1320 mc	1290 mc
b.	1330 mc	1340 mc	1290 mc
c.	1310 mc	1340 mc	1300 mc
d.	1330 mc	1340 mc	1310 mc
e.	1300 mc	1320 mc	1310 mc
f.	1330 mc	1320 mc	1310 mc
g.	1330 mc	1320 mc	1340 mc
h.	1285 mc	1345 mc	1255 mc
i.	1300 mc	1315 mc	1270 mc

For each frequency assignment, interference measurements are to be made for each at the enumerated transmitting, receiving, and antenna orientation conditions shown in the following table. Measurements are to be made with each transmitter on singly and then with both on simultaneously.

5. Interference Measurements - Data and Results

At the time of this report, no data is yet available from the interference measurements portion of the field test.

TABLE IX

Antenna Orientations

	Transmit or Receive			Antenna Orientation			Measure Interference at
	T_1	T_2	T_3	T_1	T_2	T_3	
1.	Trans	Trans	Rec. only	Point at T_3	Point at T_3	Rotate	R_3
2.	Trans	Trans	Rec. only	Point at T_3	Rotate	Point at T_1	R_3
3.	Trans	Trans	Rec. only	Rotate	Point at T_3	Point at T_1	R_3
4.	Rec. only	Trans	Trans	Rotate	Point at T_1	Point at T_1	R_1
5.	Rec. only	Trans	Trans	Point at T_3	Rotate	Point at T_1	R_1
6.	Rec. only	Trans	Trans	Point at T_3	Point at T_1	Rotate	R_1
7.	Trans	Rec. only	Trans	Rotate	Point at T_1	Point at T_2	R_2
8.	Trans	Rec. only	Trans	Point at T_2	Point at T_3	Rotate	R_2
9.	Trans	Rec. only	Trans	Point at T_2	Rotate	Point at T_2	R_2
10.	Trans	Rec. only	Trans	Rotate	Rotate	Rotate	R_2

VII. CONCLUSIONS

A number of different interference control methods have been reviewed and evaluated for the purpose of selecting one for further study, for the development of related optimization techniques and for field testing on the project. The methods considered are transmitter power control, geographical location, antenna pattern control and frequency assignments. Concurrently a review was made of techniques of operations research applicable to the control methods under consideration. The control method selected for study is frequency assignment and an optimization technique has been developed for determining the frequency assignment which results in the least interference. The most directly applicable technique here is a restricted search employing mathematical programming principles to obtain an optimum assignment.

A field test has been devised to test the validity of the assumptions made in setting up the mathematical model of the system to be optimized, and to determine whether or not the equipment characteristics are known and measured with sufficient accuracy to meet the requirements inherent in applying an optimization technique to frequency assignment problems. The field test has been initiated at the Verona Test Site, using L-band radars. The general measurements portion of the test is virtually completed at this time, and the data presently available has been compiled in this report. The data obtained so far are those needed for prediction purposes and not already available from the literature. Subsequent measurements will determine the actual amount of interference present among the radars in the test, measured in three different ways - subjective determination by an operator, measurement based on the PPI scope presentation and a count of the number of interference pulses exceeding a given threshold. When this is done for a number of frequency assignments, it will provide an experimental check on the interference prediction made and the degree of optimization achieved in the given situation.

VIII. IDENTIFICATION OF KEY PERSONNEL

The following is a listing of key personnel who have contributed to this program up to the present, together with the approximate number of hours each has spent on the program.

Name	Hours
J. E. Bridges, Manager	76
F. C. Bock	152
B. Ebstein	624
T.A. Jackson	112
A. W. Olson	271

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION
of Illinois Institute of Technology

APPROVED:

J. E. Bridges, Manager
Electronic Compatibility

B. Ebstein
Associate Engineer

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APPLICATION OF OPERATIONS RESEARCH TO INTERFERENCE, Technical Note No. 1, by F. C. Bock, B. Ebstein and A. W. Olson. October 1962. 64p. incl. illus. tables, 8 refs.
(Proj. 4540; Task 454002) (ARF 5177-TN-1; RADC-TDR-62-550)
(Contract AF 30(602)-2667) Unclassified Report

A summary of work performed during the first six months of the program is presented, encompassing a review of several interference control methods and optimization techniques; selection of a control method for further investigation; development of applicable optimization techniques; and the design of a field test to check the assumptions and input data accuracies required for the control method. Controlling the operational parameters of frequency, transmitter power, geographical location and antenna orientation has been considered, with frequency assignment the interference control method selected for further study. The mathematical tools investigated in relation to their applicability to the control methods considered are linear programming, dynamic programming, and network and combinatorial search techniques. Emphasis is placed on radar interference problems and a field test using three L-band radars is described. Sample interference minimization problems with frequency as the controlled factor are presented, as well as the data available from the initial portions of the field test.

UNCLASSIFIED

1. Radio frequency interference
2. Radar interference
3. Radar receivers
4. Radar transmitters
5. Search radars
6. Radar pulses
7. Operations research
8. Combinatorial analysis
9. Mathematical prediction
10. Digital computers
11. Programming

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